

$$E^T A E = \Lambda E^T$$

300차원 $\rightarrow$ 50차원
 차원 축소

Inclass 19: Dimensional Reduction

[SCS4049] Machine Learning and Data Science

Seongsik Park (s.park@dgu.ac.kr)

School of AI Convergence & Department of Artificial Intelligence, Dongguk University

SVD
 $\Downarrow$

주
Principal Component (PCA)
 성분
 $\nearrow$ analysis 방법.
nonnegative matrix
 $\nearrow$ factorization
 (NMF).

Principal component analysis (PCA)

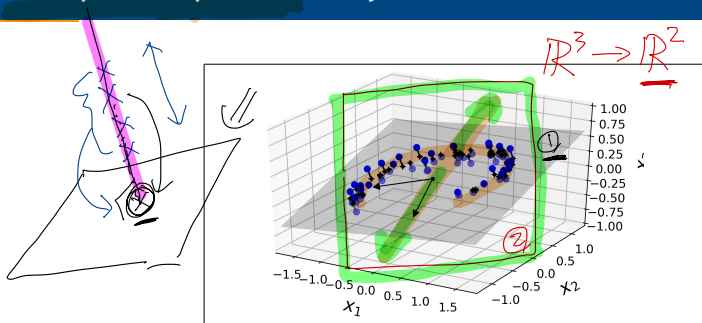


Figure 8-2. A 3D dataset lying close to a 2D subspace

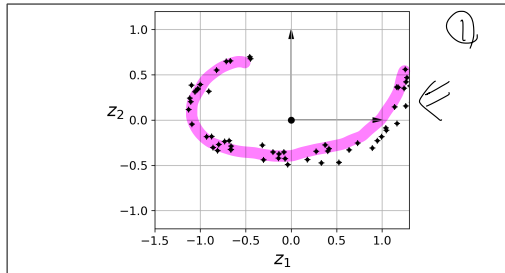
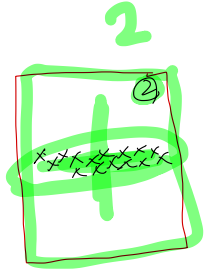


Figure 8-3. The new 2D dataset after projection

① vs ②



Principal component analysis (PCA)

Covariance measures the strength of the linear relationship between two variables

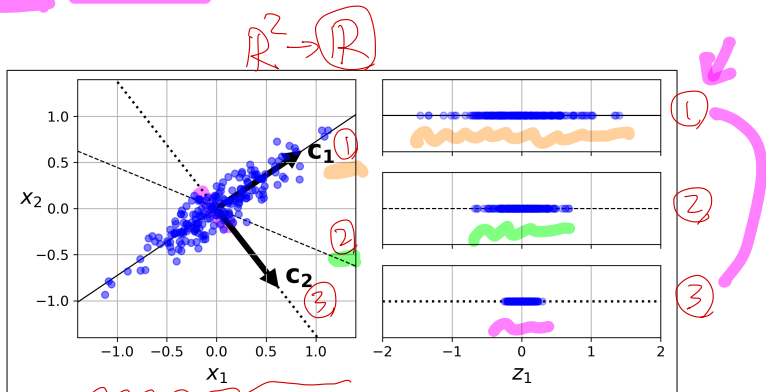
$$\sigma_{xy} = \mathbb{E}[(x - \mu_x)(y - \mu_y)] \quad (1)$$

Covariance matrix **C** for multivariate random variable **x**

$$C_{ij} = \mathbb{E}[(x_i - \mu_i)(x_j - \mu_j)] \quad (2)$$

Principal component analysis (PCA)

Preserving the variance



Principal component analysis (PCA)

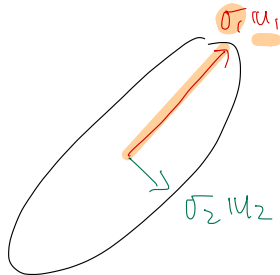
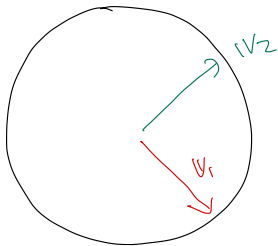
For given data $x_1, x_2, \dots, x_N \in \mathbb{R}^D$ 필요한 것

1. create a matrix $\mathbf{X} \in \mathbb{R}^{D \times N}$ with one column vector per each sample
2. covariance matrix $\mathbf{C} = \mathbb{E}[(X - \mathbb{E}(X))(X - \mathbb{E}(X))^T] \in \mathbb{R}^{D \times D}$
3. find singular vectors and singular values of $\mathbf{C} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$
4. principal components = largest singular values and vectors

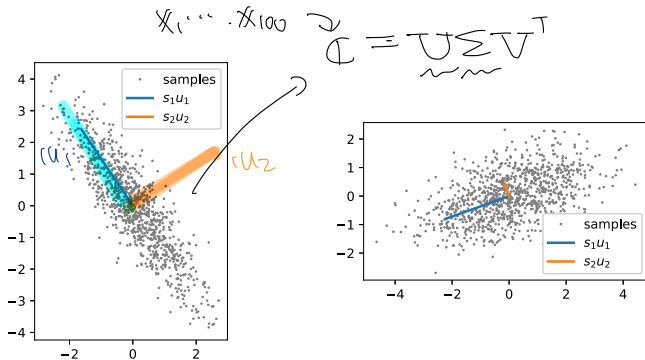
$$C_{ij} = \frac{1}{N} \sum_{n=1}^N (x_i^n - \mu_i) (x_j^n - \mu_j)$$

$$\mathbf{C} = \mathbf{C}^T \Rightarrow \mathbf{U} = \mathbf{V}$$

$$X \longrightarrow \boxed{A} X \longrightarrow$$



Principal component analysis (PCA)



Matrix factorization

- Principal component analysis (PCA): orthogonal property
- Vector quantization (VQ): unary property
- Non-negative matrix factorization (NMF): non-negativity

PCA: $\mathbf{X} \approx \mathbf{W}\mathbf{H}$

VQ: $\mathbf{X} \approx \mathbf{W}\mathbf{H}$

NMF: $\mathbf{X} \approx \mathbf{W}\mathbf{H}$

constraint.

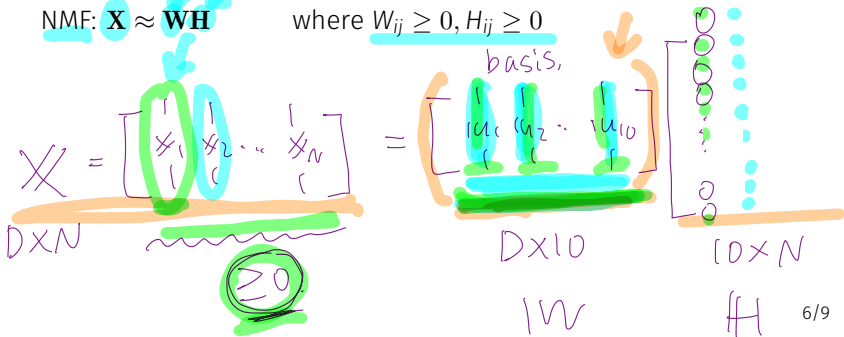
where $\mathbf{W}^T \mathbf{W} = \mathbf{I}$

$\mathbf{W} \Sigma \mathbf{V}^T$

where $\mathbf{H}$ consists of unary vectors

weight.

where $W_{ij} \geq 0, H_{ij} \geq 0$

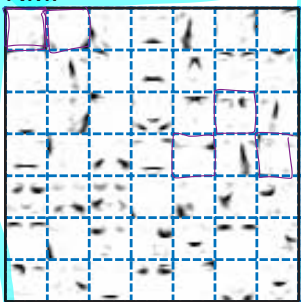


Matrix factorization and face image compression

경우 0
5
필수 1.

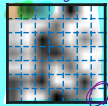
NMF

49 basis



x

weight



=



Original



1000

VQ



x



||



PCA



x



||



필수 1, -1
경우 0
필수 1

Nonnegative matrix factorization (NMF)

Nonnegative matrix factorization (NMF)

optimization

given $\mathbf{X} \in \mathbb{R}^{n \times m}$
 $\Rightarrow$ find $(\mathbf{W}, \mathbf{H}) = \arg \min_{(\mathbf{W}, \mathbf{H})} \|\mathbf{X} - \mathbf{WH}\|_F^2$

$\mathbf{X} \approx \mathbf{WH}$

$\Rightarrow$ s.t. $\mathbf{W} \in \mathbb{R}_+^{n \times k}$ and $\mathbf{H} \in \mathbb{R}_+^{k \times m}$ ($k \leq \text{rank}(\mathbf{A})$)

$$\mathbb{R}_+ = \{x \geq 0\}$$

$\rightarrow$ ~~closed~~ approx $(\mathbf{W}, \mathbf{H}) = \text{np.nmf}(\mathbf{X})$

PCA vs. NMF for MNIST dataset

